The Economic Value of TIPS Arbitrage Mispricing*

Shehryar Amin,† Vasilis Dedes,‡ Roméo Tédongap ,§ and Bingxin Xing ¶

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Abstract

Rational frictionless asset pricing models imply that continuously compounded zero-coupon inflation swap and break-even inflation rates with same maturity must be equal. The data, however, evidence a persistent positive difference between these two quantities, which the literature attributes to mispricing of Treasury Inflation-Protected Securities (TIPS). In theory, factors driving TIPS mispricing are not directly observable to the econometrician. To reveal these factors, we analyze the daily term structure of TIPS mispricing and uncover its information content. To assess its economic value, we derive novel high-frequency stylized facts about its dynamics. In particular, we document strong relationships with stock market returns, option-implied volatility and variance risk premium, and an important channel for predicting inflation, bond and equity excess returns, jointly.

JEL Classification Codes: G11, G12

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†ESSEC Business School; shehryar.amin@essec.edu.

‡BlackRock, Inc; vasilis.dedes@blackrock.com.

§ESSEC Business School; tedongap@essec.edu.

¶ESSEC Business School; bingxin.xing@essec.edu.

1 Introduction

Financial markets are subject to incompleteness and frictions that may induce abnormal price deviations from their equilibrium, fundamental, or fair values. If such price deviations are observable and quantifiable, they lead to arbitrage opportunities. In efficient markets, these arbitrage opportunities would generate profits only almost immediately, then disappear quickly once subsequent variations in supply and demand have restored prices to their equilibrium values. In practice, however, mispricing and induced profitable trading strategies may remain and may not be fully eliminated by arbitrage. That's because in real markets, arbitrage is in general risky and costly, and the number of informed arbitrageurs or the supply of capital they have to invest to exploit mispricing is limited (Emmons and Schmid, 2002, Shleifer and Vishny, 1997). In fixed income markets, observable mispricing varies across the maturity of the payoff, and the goal of this article is to examine its term-structure and reveal its economic content.

We build on the recent and growing literature on one of the most pervasive arbitrage mispricings documented in Treasury bond markets. The continuously compounded zero-coupon inflation-protected and nominal bond yields, and the inflation swap rate of same maturity, are tied together by a no-arbitrage restriction, implying that inflation swap and break-even inflation rates are equal. Data evidence, however, that the inflation swap rate is almost always, regardless of maturity, greater than the break-even inflation rate. The literature attributes this consistent positive difference between inflation swap and break-even inflation rates mainly to the mispricing of Treasury Inflation-Protected Securities (TIPS), in particular, due to illiquidity in the TIPS market (see, for example, Fleckenstein et al., 2014, Haubrich et al., 2012, and Christensen and Gillan, 2012). [drop this: To the contrary of the existing literature, we do not attempt to explain TIPS arbitrage mispricing but instead, given its persistence, variation both across time and maturity, and strong commonality across the maturity dimension, we analyze its term structure and study its informational content.]

We proceed in several steps. First, we use daily inflation swap and break-even inflation rates from January 2005 to December 2019 to construct TIPS mispricing across 12 maturities ranging from 2 to 20 years. The term-structure of average TIPS arbitrage mispricing is nontrivial, with a pronounced hump shape, the five-year maturity appearing to be the most mispriced on average. We show that the term-structure of TIPS mispricing exhibits a low-dimensional factor structure. Its first three principal components explain about 97% of total variation. Each component has a systematic effect across maturities and they can be interpreted as level, slope and curvature factors, respectively.

Second, we examine dynamic daily cross-correlations of TIPS mispricing with leads and lags ranging up to 90 days of stock market options-implied volatility (VIX) and variance risk premium (VRP). This analysis is motivated by the fact that the difference between inflation swap and break-even inflation rates has the same properties as volatility measures including positivity, positive skewness, and significant excess kurtosis. We find striking patterns. In particular, the cross-correlations of leads and lags of the squared option-implied VIX volatility index with the level factor of TIPS mispricing are positive and exhibit an inverted U shape with the daily horizon, ranging between 0.40 and 0.80. In contrast, the correlations between the slope factor of TIPS mispricing and lagged squared VIX are negative, lasting for several days. They are decreasing in magnitude from -0.33 to zero for lags of the squared VIX shorter than about 30 days, and remain closer to zero for lags longer than about 30 days. On the other hand, the correlations of the slope factor with future squared VIX are negative and closer to -0.40.

Next, we examine the information content of the term-structure of TIPS mispricing in predicting inflation and excess returns. Arbitrage-free affine models (Duffie et al., 2000; Lettau and Wachter, 2011) imply that expected inflation and excess returns are linear functions of the same risk factors. In other words, inflation forecast and risk premia should exhibit a factor structure. In general, these factors are unobservable to the econometrician. Nonetheless, we assume that the risk factors form a basis for the term structure of TIPS mispricing. In particular, a small number of linear combinations of TIPS mispricing across the maturity spectrum should predict inflation and excess returns.

We estimate how many factors from the term structure of TIPS mispricing are sufficient to summarize its predictive content for inflation, and bond and equity excess returns jointly. We use the robust procedure of Cook and Setodji (2003). This dimension-reduction procedure does not focus a priori on the leading principal components. The test does not rely on any distributional assumptions. It is also robust to departures from linearity. We find that four factors are sufficient to summarize the joint predictability of inflation, and bond and equity excess returns across maturities and across horizons. A detailed analysis reveals two common factors for bond and equity premia, one bond risk premia specific factor, and one equity risk premia specific factor.

We find that TIPS mispricing predicts jointly inflation, and bond and equity excess returns. The predictive content of TIPS mispricing is stable when we predict bond and equity excess returns separately. We conclude that the predictive content of TIPS mispricing is also robust when we account for the variance risk premium.

The literature on mispricing is vast, and covers several asset classes including equities (Brennan and Wang, 2010; Kapadia and Pu, 2012; Lamont and Thaler, 2003; Sadka and Scherbina, 2007), currencies (Akram et al., 2008; Coffey et al., 2009; Fong et al., 2010; Kozhan and Tham, 2012; Lyons and Moore, 2009; Mancini-Griffoli and Ranaldo, 2011; Marshall et al., 2013; Pasquariello and Zhu, 2011; Pasquariello, 2014; Pasquariello, 2015), bonds (Bretscher, 2014; Buraschi et al., 2013; Chan and Chen, 2007; Dick-Nielsen and Rossi, 2013; Fleckenstein et al., 2014; Krishnamurthy, 2002; Longstaff, 1992; Lou et al., 2013; Musto et al., 2015), and CDS (Bai and Collin-Dufresne, 2013; Duarte et al., 2007; Duffie, 2010; Fontana, 2011; Mitchell and Pulvino, 2012; Nashikkar et al., 2011; Stanton and Wallace, 2011), just to name a few.[1]

2 TIPS arbitrage mispricing

Denote by $M^{\$}_{t,t+1}$ the pricing kernel used to value nominal payoffs between dates t and $t+1$, and let I_t denote the level of the price index at date t. Denote by $\pi_{t+1} \equiv \ln I_{t+1} - \ln I_t$ the continuously compounded realized inflation rate between date t and date $t+1$.

2.1 Break-even inflation

By definition, the n-period continuously compounded zero-coupon inflation-protected and nominal bond yields, denoted $y_{n,t}$ and $y^{\$}_{n,t}$ respectively, are given by

$$y_{n,t} = -\frac{1}{n} \ln \mathbb{E}_t \left[M^{\$}_{t,t+n} \exp\left(\pi_{t-k,t+n-k}\right) \right] \quad \text{and} \quad y^{\$}_{n,t} = -\frac{1}{n} \ln \mathbb{E}_t \left[M^{\$}_{t,t+n} \right], \tag{1}$$

where

$$M^{\$}_{t,t+n} \equiv \prod_{j=1}^{n} M^{\$}_{t+j-1,t+j} \quad \text{and} \quad \pi_{t,t+n} \equiv \sum_{j=1}^{n} \pi_{t+j} = \ln I_{t+n} - \ln I_t \tag{2}$$

are the n-period nominal pricing kernel and realized inflation rate between dates t and $t+n$, respectively, and $\mathbb{E}_t[\cdot]$ denotes the real-world conditional expectation operator. The formula of the zero-coupon inflation-protected bond yield accounts for the indexation lag k, as in practice, the payoff is not fully protected against inflation. For example, the inflation index for a TIPS payment is based on the consumer price index (CPI) recorded about three months prior to the payment date.

The difference $y^{\$}_{n,t} - y_{n,t}$ corresponds to break-even inflation rate, which by using the defini-

[1] For theoretical work see, for example, Brunnermeier and Pedersen (2009), Duffie (2010), Gromb and Vayanos (2002), and Shleifer and Vishny (1997).

tions of inflation-protected and nominal bond yields in Equation (1) may be written

$$y^{\$}_{n,t} - y_{n,t} = \frac{1}{n} \ln \mathbb{E}^{\mathbb{Q}(n)}_t \left[\exp\left(\pi_{t-k,t+n-k}\right)\right], \tag{3}$$

where $\mathbb{E}^{\mathbb{Q}(n)}_t[\cdot]$ denotes the n-period risk-neutral conditional expectation operator defined by

$$\mathbb{E}^{\mathbb{Q}(n)}_t\left[X_{t+n}\right] \equiv \mathbb{E}_t\left[\frac{M^{\$}_{t,t+n}}{\mathbb{E}_t\left[M^{\$}_{t,t+n}\right]} X_{t+n}\right], \tag{4}$$

with X_{t+n} being a quantity known at time $t+n$.

2.2 Inflation swap

A zero-coupon inflation swap is a derivative contract in which the inflation buyer agrees to swap a fixed payment for a floating payment linked to the inflation rate, for a given notional amount and period of time. The continuously compounded inflation swap rate, $s_{n,t}$, is the fixed interest rate which at the initiation date equates the fixed leg and the floating leg of the swap contract. In practice, the floating leg of inflation swap payments uses the same indexation lag as for TIPS payments. Formally we have at inception that

$$\underbrace{\mathbb{E}_t\left[M^{\$}_{t,t+n}\exp\left(ns_{n,t}\right)\right]}_{fixed\ leg} = \underbrace{\mathbb{E}_t\left[M^{\$}_{t,t+n}\exp\left(\pi_{t-k,t+n-k}\right)\right]}_{floating\ leg} \tag{5}$$

implying that

$$s_{n,t} = \frac{1}{n}\ln \mathbb{E}_t\left[\frac{M^{\$}_{t,t+n}}{\mathbb{E}_t\left[M^{\$}_{t,t+n}\right]}\exp\left(\pi_{t-k,t+n-k}\right)\right], \tag{6}$$

and consequently we have $s_{n,t} = y^{\$}_{n,t} - y_{n,t}$. Hence, inflation swap and break-even inflation rates are the same quantity. However, this equality does not hold in the data, which evidence a consistent positive difference between inflation swap and break-even inflation rates.

2.3 Arbitrage

Suppose that TIPS are mispriced, typically undervalued, and let $d_{n,t}$ denote the difference between the observed n-period zero-coupon inflation-protected bond yield, $\hat{y}_{n,t}$, and its arbitrage-free counterpart, $y_{n,t}$. We have

$$\begin{aligned} d_{n,t} &\equiv \hat{y}_{n,t} - y_{n,t} \\ &= s_{n,t} - \left(y^{\$}_{n,t} - \hat{y}_{n,t}\right). \end{aligned} \tag{7}$$

Suppose you observe $d_{n,t} > 0$ at date t, and consider the following strategy:

> Borrow n-period zero-coupon nominal bonds for a par value of \$100 and sell them for \$$100P^{\$}_{n,t}$; use the proceeds to buy n-period zero-coupon inflation-protected bonds for a par value of \$$100\exp\left(-ns_{n,t}\right)$, at the price \$$100\exp\left(-ns_{n,t}\right)\hat{P}_{n,t}$, and enter an inflation swap contract with notional amount equal to \$$100\exp\left(-ns_{n,t}\right)$, as fixed rate receiver.

At date $t+n$, pay \$100 for the nominal bonds, get \$$100\exp\left(-ns_{n,t}\right)\exp\left(\pi_{t-k,t+n-k}\right)$ from the inflation-protected bonds, and receive a net payment from the inflation swap that is equal to \$$100\exp\left(-ns_{n,t}\right) \times \left[\exp\left(ns_{n,t}\right) - \exp\left(\pi_{t-k,t+n-k}\right)\right]$. Thus, the net gain at date $t+n$ is null. At date t however, the difference \$$100P^{\$}_{n,t} -$ \$$100\exp\left(-ns_{n,t}\right)\hat{P}_{n,t}$, which is also equal to \$$100\exp\left(-n\left(s_{n,t} + \hat{y}_{n,t}\right)\right)\left[\exp\left(nd_{n,t}\right) - 1\right]$, is positive and represents an arbitrage profit. Following Fleckenstein et al. (2014), we refer to this quantity as the dollar mispricing, and to $d_{n,t}$ as the basis point mispricing.

The current study will focus on analyzing the information content of the term structure of basis point mispricing. We derive a collection of stylized facts about the term structure of $d_{n,t}$, using data on zero-coupon inflation-protected and nominal bond yields as well as inflation swap rates. Although the literature that uses inflation swap and break-even inflation rates is growing, the empirical facts of these time series are not well-established in a methodical way. One of the main contributions of this paper is to provide a set of stylized facts on these market-based measures of inflation expectations. Another contribution is to investigate the term structure of TIPS arbitrage mispricing.

Fleckenstein et al. (2014) address the question whether the mispricing in TIPS is due to a risk premium. They argue that since the mispricing in TIPS is a violation of an arbitrage-free relationship (law of one price), it cannot be reconciled with an equilibrium asset pricing model. They conclude, however, that such violations of arbitrage-free relationships may be referred to in the literature as liquidity risk premia, liquidity effects, etc. In fact, in many studies (see, for example, Abrahams et al., 2015, Christensen and Gillan, 2012, D'Amico et al., 2014, Grishchenko and Huang, 2013, Haubrich et al., 2012) the term liquidity risk premium is used to capture the mispricing in the TIPS. This is mainly done in order to correct TIPS prices for the fact that they are relatively illiquid compared to the Treasury bond. In the context of these studies, this results in measuring inflation risk premia which are adjusted for the illiquidity in the TIPS market. Nevertheless, liquidity is one of the main drivers of this violation (see Fleckenstein et al., 2014), among other things. Therefore, the information content of the TIPS mispricing term structure may reflect liquidity information as well.

3 Data

We use daily data on U.S. nominal Treasury yields, U.S. Treasury inflation-protected securities (TIPS) yields, and inflation swap rates from January 2005 to December 2019. Zero-coupon yields on nominal Treasury bonds and TIPS are constructed by Gürkaynak et al. (2010).[2] Both yield curves are constructed assuming that the instantaneous forward rates follow a Nelson-Siegel-Svensson functional form. This parametric specification results in a smooth yield curve, which is estimated by choosing the parameters to minimize the weighted

[2] Zero-coupon yields on nominal Treasury bonds were originally constructed by Gürkaynak et al. (2007). The same methodology is applied by Gürkaynak et al. (2010) to construct zero-coupon yields on TIPS.

sum of the squared deviations between actual and predicted prices of Treasury securities. Nevertheless, the fitting errors are very small, thus, we assume that the fitted yields by Gürkaynak et al. (2010) correspond to the actual yields.

Gürkaynak et al. (2010) report zero-coupon yields on TIPS for maturities between 2 and 20 years based on the available range of maturities on the underlying quotes on TIPS used on the yield curve estimation.[3] Zero-coupon inflation swap rates are obtained from Bloomberg. The traded maturities are between 1 and 10 years, as well as 12, 15, 20, 25, and 30 years. Nevertheless, we restrict our sample to those maturities which are available for TIPS. The reference CPI for both TIPS and inflation swaps is the non-seasonally adjusted CPI-U for the third preceding calendar month.

Fleming and Sporn (2013) find that the average difference between inflation swap rates from actual transactions and those quoted from Bloomberg is less than 1 basis point.[4,5] Hence, we assume that $d_{n,t}$ captures only the difference between the observed n-period zero-coupon real yield, $\hat{y}_{n,t}$, and its arbitrage-free counterpart, $y_{n,t}$, as shown in Equation (7), which allows us to study the term structure of TIPS mispricing at daily frequency.

In addition, we use daily data on VIX, which are obtained from Bloomberg. We obtain the Fama-Bliss zero-coupon yields from CRSP in order to construct excess returns on bonds. Unlike the smooth fitted yield curve using a parametric specification, the yield curve construction by Fama and Bliss (1987) produces unsmoothed zero-coupon yields which exactly price the included bonds.[6] Excess equity returns are constructed using data obtained from Kenneth French's data library. All excess returns are available at the monthly frequency.

[3]Gürkaynak et al. (2007) and Gürkaynak et al. (2010) report the estimated parameters from the Nelson-Siegel-Svensson yield curve in addition to the nominal and real zero-coupon yields. Thus, we can construct zero-coupon yields for any given horizon. However, Gürkaynak et al. (2007) strongly recommend only focusing at the horizons for which outstanding securities were available for estimation.

[4]Bloomberg quotes are end-of-day midquotes.

[5]Fleming and Sporn (2013) use data on actual transactions from MarkitSERV. The average difference between MarkitSERV and Bloomberg quotes is 0.8 basis points and the standard deviation is 3.7 basis points, based on 107 new transactions between June 1 and August 31, 2010.

[6]The Fama-Bliss data are standard in the literature of predictability in bond returns. We use these data in order to make our results comparable to those obtained in previous studies.

Table 1 shows summary statistics of inflation swap rates, break-even inflation rates, and their difference across maturities. Inflation swap and break-even inflation rates are persistent with 22-day autocorrelation coefficients ranging between 0.809 and 0.908 across maturities. Their difference, namely the TIPS arbitrage mispricing, is less persistent with 22-day autocorrelation coefficients ranging between 0.582 and 0.866 across maturities. Inflation swap and break-even inflation rates are strongly negatively skewed. However, TIPS arbitrage mispricing exhibits a large positive skewness, mainly due to the fact that the break-even inflation rate is consistently more negatively skewed than the actual inflation swap rate, suggesting that extreme undervaluation of TIPS relative to nominal bonds is more likely than their extreme overvaluation. All three time series exhibit significant excess kurtosis. For the TIPS mispricing in particular, excess kurtosis is about 10 and higher for maturities from 4 to 10 years. The TIPS arbitrage mispricing is strictly positive most of the time, again reflecting that TIPS are typically underpriced relative to nominal bonds. The maturity with the most negative observations is the two-year, and TIPS mispricing for maturities between 5 and 20 years is strictly positive at least 98.6% of the time in our sample period.[7]

Figure 1 shows the time series of inflation swap rates, break-even inflation rates, and their difference for various maturities. Break-even inflation rates are more volatile than inflation swap rates, particularly for the short-term maturities. The time series of the difference also shows significant variation over time. The financial crisis in 2008 resulted in a number of changes on both the level and the term structure of these time series. Prior to the crisis, inflation swap rates and break-even inflation rates fluctuated roughly between 2% and 3%, while during the crisis both quantities dropped sharply. Inflation expectations over longer time horizons dropped to a level of about 1% and the the two-year expected inflation

[7] See Fleming and Sporn (2013) for a study of trading activity and price transparency in the U.S. inflation swap market. In Chart 2 they report inflation swap trading activity. The maturity with the highest activity is the ten-year, followed by the five-year and the three-year, and the maturities with the lowest activity are the twelve-year and thirty-year, followed by the two-year. In Chart 3 they report inflation swap trade size. The maturity with the largest mean trade size is the one-year, followed by maturities of 4 years, 3 years, 2 years, and 8 years, and the maturity with the smallest mean trade size is the five-year, followed by the two-year.

far below -3%. This reflects the fears of a meltdown in the economy and the subsequent deflationary concerns. Nevertheless, before the end of 2009 both measures of market-based inflation expectations rose above 0%. Remarkably, the expectations of inflation in the long-run reverted to their prior mean within less than a year, exhibiting similar dispersion as before the crisis. Whereas, the two-year expected inflation has remained far below its prior mean level and has become more volatile.

Regarding the changes in the shape of the term structure of market-based inflation expectations, the slope after the financial crisis has remained quite steep, indicating that markets' deflationary concerns over a short time horizon remain considerable, although from 2014 onwards a flattening of the curve is observed. The parallel shift of the curve downwards during the second half of 2014 is due to the falling oil prices, which has raised deflationary concerns even over longer time horizons.

The term structures of mean, median, and standard deviation of inflation swap rates, break-even inflation rates, and their difference are illustrated in Figure 2. The term structure of the mean of both market-based measures of inflation expectations is upward sloping and both series exhibit decreasing dispersion (standard deviation is downward sloping) as the maturity increases. The latter observation indicates that markets expect inflation to be mean reverting in the long-run. The term structure of TIPS mispricing, which is the difference between the two rates, is upward sloping at the short end, it peaks at the five-year maturity at 33.3 basis points, and it becomes downward sloping for the remaining maturities up to 15 years.

4 The term structure of TIPS arbitrage mispricing

The bottom panel of Figure 1 provides evidence of strong commonality across the term structure of TIPS arbitrage mispricing, suggesting that a few systematic factors may explain the variation in the term structure of TIPS arbitrage mispricing. The top and the middle panels of Figure 1 also suggest that the strong comovement across the term-structure of TIPS arbitrage mispricing is a consequence of similar correlations across maturities of inflation swap and break-even inflation rates. To further investigate this factor structure, we run a principal component analysis (PCA) on the correlation matrix of all available maturities of TIPS arbitrage mispricing. The PCA reveals that the first three principal components explain 96.9% of the total variation at all available maturities.

The results from the PCA on TIPS arbitrage mispricing across maturities are reported in Figure 3. The scores (right panel) exhibit significant time variation throughout the sample period, and the patterns of loadings on the first three principal components across maturities (left panel) suggest the usual level, slope, and curvature interpretation from the yield curve literature. This interpretation is confirmed by Table 2 which reports the sample correlation between the first three principal components and actual measures of the level, slope, and curvature of the term structure of TIPS arbitrage mispricing. The correlation between the first principal component and the actual level, average of $d_{n,t}$ across maturities, is 0.995; the correlation between the second principal component and the actual slope, the difference $d_{20,t} - d_{2,t}$, is 0.716; and the correlation between the third principal component and the actual curvature, the quantity $d_{20,t} + d_{2,t} - 2d_{8,t}$, is 0.932.

Since TIPS arbitrage mispricing shares very similar properties with measures of stock market variation including positivity, positive skewness and significant excess kurtosis as shown in the bottom panel of Table 1, we also report the sample correlations of the first three principal components with various measures of stock market variation, and the variance risk premium.[8] In particular, the first principal component shows strong positive correlations with proxies of risk-neutral variance (VIX^2), real-world variance (RVFor), risk-neutral volatility (VIX), and realized variance (RV); similar correlations values are obtained for the actual level of the TIPS mispricing curve. The intuition for this result is the following. In times of high stock market uncertainty, investors sentiment deteriorates and arbitrage is mainly perceived

[8]The variance risk premium is defined as the difference between the expected future variations under the risk-neutral and the real-world probability measures.

as a risky and costly strategy (see Shleifer and Vishny, 1997).[9] In consequence, attempts to profit from mispricing are reduced, and TIPS for all maturities remain at higher underpricing levels relative to corresponding nominal bonds. Once stock market uncertainty is progressively resolved and reduced, investors gain confidence and profitable trading strategies are implemented. This helps to reduce the degree of TIPS arbitrage mispricing, but not entirely due to slow-moving capital. The correlation of the first principal component with the risk-neutral variance (0.735) is higher than with the real-world variance (0.638), also suggesting that the level of the TIPS mispricing curve has a forward-looking character.

While stock market uncertainty affects the level of TIPS mispricing, this effect is asymmetric across maturities and thereby it affects the slope of TIPS mispricing as well. The second principal component is significantly negatively correlated with the same measures of stock market variation; the correlation values are stronger for the actual slope of the TIPS mispricing curve. Suppose stock market uncertainty increases and that the level TIPS mispricing increases; if there were any attempt to profit from mispricing by implementing an arbitrage strategy, investors would favor long-maturity TIPS relative to short-maturity ones, thus flattening the term structure of TIPS mispricing. This is because increasing stock market uncertainty may be associated with increasing macroeconomic uncertainty. High persistence in macroeconomic uncertainty can lead to extended periods of slow real growth over the long run, and since the arbitrage strategy implies holding the TIPS as described in Section 2.2, it would also provide a hedge against long-run consumption risk.

We also examine correlations between systematic factors driving the TIPS mispricing curve and the variance risk premium. The level factor is strongly positively correlated with the variance risk premium (0.667), while the slope factor is significantly negatively correlated with the variance risk premium (-0.418). Since the variance risk premium has been shown

[9] See also Liu and Longstaff (2004) who show that a textbook arbitrage opportunity may be risky and result in losses when investors do not have sufficient collateral to meet margin calls and be forced to liquidate their positions, and Basak and Croitoru (2000) who show that mispricing may arise in general equilibrium as a result of heterogeneous agents facing portfolio constraints.

to be an important short-term predictor of returns (see for example Bollerslev et al., 2009), these correlations also suggest that there may be a link between TIPS mispricing and return predictability, a feature that we examine empirically in Section 6.

5 TIPS arbitrage mispricing and financial volatility

To further investigate the empirical patterns of TIPS arbitrage mispricing and its relationship with financial volatility, in this section we analyze the sample autocorrelations and dynamic cross-correlations of the principal components with measures of stock market variation, variance risk premium, and returns, based on daily observations.

We start by establishing the link between TIPS mispricing and the options-implied volatility index VIX. The VIX index represents the market's options-implied expectation of the cumulative variation of the S&P 500 index over the next month plus a potential variance risk premium for bearing the corresponding volatility risk. The top panels of Figure 4 show the two- and five-year maturity TIPS mispricing (left y-axis) and the VIX volatility index (right y-axis). TIPS mispricing and VIX exhibit very similar patterns in their time series. The bottom panels of Figure 4 show sample autocorrelations and dynamic cross-correlations between TIPS mispricing and VIX to a lag length of 90 days. The autocorrelations of TIPS mispricing (black line) and VIX (green line) decay at a slow rate and have similar patterns. The blue lines represent the cross-correlations between leads and lags of VIX, which are positive and exhibit and inverted U shape, ranging between 0.30 and 0.80.

We turn next to the relation between the principal components and measures of stock market variation, variance risk premium, and returns. The left panel of Figure 5 plots the squared option-implied VIX volatility index and the one-month ahead forecast of realized variance constructed from a simple autoregressive-type model for the daily realized volatilities, the Heterogeneous Autoregressive model of the Realized Volatility (HAR-RV) proposed by Corsi (2009), as in Bollerslev et al. (2012) and Bonomo et al. (2015). Bollerslev et al. (2012) use

VIX^2 to estimate the risk-neutral expectation of the forward integrated variance in a model-free fashion. The one-month ahead forecast of realized variance is used to estimate the forward integrated variance under the objective measure. The variance risk premium is defined as the difference between the risk-neutral and objective expectations of the forward integrated variance. The right panel of Figure 5 plots the daily time series of the variance risk premium. The sample mean of the variance premium is 19.05 basis points and the sample standard deviation is 27.01 basis points.

In Figure 6 we plot the sample autocorrelations to a lag length of 90 days. The VIX and the first principal components are the most persistent series and their autocorrelation patterns are almost identical for all lags. This pattern suggests that the level of the TIPS mispricing term structure is a highly persistent process and its autocorrelation has a rate of decay which is typical of a volatility process. This result confirms the indirect evidence that Fleckenstein et al. (2014) find for the persistence of mispricing in the TIPS market. Our direct empirical evidence of persistence in the level of TIPS mispricing, and the fact that the level has similar characteristics with stock market variation, is also consistent with the theoretical framework of Shleifer and Vishny (1997). High uncertainty in financial markets, makes arbitrage less attractive to risk-averse arbitrageurs, whose trading activity will bring mispriced assets back to fundamental values. The persistence in uncertainty in financial markets results in extended periods of deteriorated arbitrage activity, and hence persistence in mispricing.

The autocorrelations of the second principal component decay at a slower rate similarly to the variance risk premium. Their pattern is also similar to the autocorrelation pattern of VIX, which suggests that the slope of the TIPS mispricing term structure is closely related to the risk premium for bearing volatility risk and to some extent to the option-implied stock market variation. The autocorrelations also indicate a faster mean reversion in the slope of the TIPS mispricing term structure than in the level, which is similar to that of the variance risk premium. Finally, the third principal component is the least persistent.

Figure 7 plots the dynamic cross-correlations between the principal components and VIX^2, VIX, variance risk premium, and returns for leads and lags ranging up to 90 days. The level of the TIPS mispricing term structure is positively correlated with leads and lags of the option-implied stock market variation and the variance risk premium. This lead-lag relationship has an inverted U shape and it peaks at about 20-day lags of the VIX^2 and the VIX, and at about 10-day leads of the variance risk premium.

The correlations between the slope of the TIPS mispricing curve and lagged VIX^2 are negative, lasting for several days. They are decreasing in magnitude from -0.33 to zero for lags of the VIX^2 shorter than about 30 days, and remain closer to zero for lags longer than about 30 days. This lead-lag relationship suggests that as option-implied stock market variation increases, TIPS mispricing curve is expected to invert (a decrease in slope results either in inversion of the curve that is on average flat or flattening of the curve that is on average up-warding sloping - TIPS mispricing curve is on average slightly downward sloping). Thus, mispricing of short-term maturity TIPS is expected to increase more relative to long-term maturities. This differential mispricing was pronounced during the financial crisis, when the two-year TIPS mispricing was about 250 basis points and the twenty-year TIPS mispricing was about 100 basis.

Recessions, in general, result in persistent low growth. The fear of an extended period of low growth would make investors during periods of increased macroeconomic uncertainty more willing to hedge against long-run consumption risks, and hence willing to buy long-term maturity real (inflation-indexed) bonds. This differential in preferences for long-term TIPS will move capital from the short-term maturities to the long-term maturities, which results in the differential mispricing across maturities. On the other hand, when macroeconomic uncertainty is resolved, which is reflected in lower expected stock market variation, capital flows are not concentrated on long-term TIPS only. Therefore, a flattening (increase of the slope) of the TIPS mispricing curve is expected in the future.

In contrast, the correlations of the slope factor with future VIX^2 are negative and stable at about -0.40. We refer to these negative correlations as the mispricing feedback effect. As investors observe an increase in differential mispricing of short-term TIPS (decrease of the slope), their fear of an extended period of macroeconomic uncertainty is reflected in higher expected stock market variation.

The cross-correlations between the curvature factor, and stock market variation and the variance risk premium have an increasing pattern from lags to leads, ranging from -0.2 to 0.2. The lead-lag relationship between the TIPS mispricing factors and returns, however, does not have a clear systematic pattern. Perhaps most interestingly, the correlations between the level and lagged returns are negative and the correlations between the level and future returns converge to zero for leads longer than two weeks. The left part of the figure in the bottom panel of Figure 7 illustrates the leverage effect: negative correlations between lagged returns and current stock market variation. The right part of the figure illustrates positive correlations between stock market variation and future returns which is referred to as the volatility feedback effect (as in Bollerslev et al., 2012).

6 TIPS arbitrage mispricing and asset risk premia

We consider the following model for predicting jointly future realizations of inflation, bond excess returns, and equity excess returns

$$Z_t = \alpha + \Pi D_t + \epsilon_t, \tag{8}$$

where in Z_t we stack one-year ahead inflation, π_{t+12}, bond excess returns $rx^b_{n,t+12}$, where $n = 2, 3, 4, 5$ years, and equity excess returns, rx^e_{t+h}, where $h = 1, 2, 3, 6, 9, 12$ months, and $D_t = \{d_{n,t}\}_{n=1,\ldots,p_1}$ is a vector of dimension p_1 with the difference between inflation swap rates and break-even inflation rates at all available maturities. Panel A of Table 3 reports the p-values of the Cook-Setodji test statistic, $\widehat{\Lambda}_m$, for ranks ranging from 0 to 10. We reject that $\text{rank}(\Pi) = 0, 1, 2, 3$, and we do not reject that $\text{rank}(\Pi) = 4$: four linear combinations of TIPS arbitrage mispricing at all available maturities predict inflation, bond excess returns, and equity excess returns, jointly.

Panel B of Table 3 reports the adjusted R^2s of predictability regressions of one-year ahead realized inflation for different ranks of matrix Π. The R^2 for $r = 4$ is 28.6%, and the R^2 for $r = 11$, which corresponds to the OLS predictability regression (matrix Π in Equation (8) has full rank) is 28.5%. Panel C of Table 3 reports the adjusted R^2s of predictability regressions of bond excess returns. The R^2s for $r = 4$ are 15.4%, 19.6%, 19.2%, 23,4% for one-year excess returns on two- to five-year bonds, respectively. Panel D of Table 3 reports the adjusted R^2s of predictability regressions of equity excess returns for one- through twelve-month horizon. The R^2s for $r = 4$ peak at the two-month horizon, and for one-, two-, three-, and six-month horizon decrease for all higher ranks. The R^2s for the nine-month and twelve-month horizon for $r = 4$ are 28.4% and 17.4%, respectively, and for $r = 11$, 26.5% and 17.9%, respectively.

In order to assess the predictive content of the TIPS mispricing factors for equity excess returns we compare our results with those obtained from return predictability regressions on the variance risk premium. Bollerslev et al. (2009) show that the difference between implied and realized variation, or the variance risk premium, explains equity returns, and find an R^2 of 6.8% for return predictability at a three-month horizon. In Table 4 we report the results of predictability regressions of equity returns on the variance risk premium for one- through twelve-month horizon. The panels, from top to bottom, correspond to the main sample, the full sample (data on VIX are available from January 1990 onwards), the sample used by Bonomo et al. (2015), and the sample used by Bollerslev et al. (2009). The results we obtain in our sample, from January 2005 to December 2019, exhibit similar patters, and comparable magnitudes with those obtained in the previous studies. The R^2 peaks at the three-month horizon and $\hat{\beta}_h$ remains significant for all horizons, but its magnitude decreases for longer horizons.

Table 5 reports the results (p-values of the Cook-Setodji test statistic in Panel A, and adjusted R^2s in Panels B and C) of predictability regressions of equity excess returns for one- through twelve-month horizon for different ranks of matrix Π. In Panel B we consider Equation (8), in which Z_t contains only equity excess returns, rx^e_{t+h}, for $h = 1, 2, \ldots, 12$ months. In Panel C we combine the difference between inflation swap rates and break-even inflation rates at all available maturities, D_t, with the variance risk premium, vrp_t,

$$Z_t = \alpha + \Pi D_t + \psi vrp_t + \epsilon_t. \tag{9}$$

We do not reject that $\text{rank}(\Pi) = 3$, thus, three risk factors summarize the predictive content of the term structure of TIPS mispricing for equity excess returns. The R^2s in Panel B have the same pattern and similar magnitude with those in Panel D of Table 3. Adding the variance risk premium in the predictability regressions results in increased R^2s. For the one- and three-month horizon R^2s increase 6.5 and 8.2 percentage points, respectively, and for the remaining horizons the increase is between 0.5 and 2.7 percentage points. The addition of the variance risk premium also shifts the peak in R^2s from two months to three months as in the return predictability regressions on the variance risk premium alone, see Table 4. Thus, the predictive content of the term structure of TIPS mispricing for equity excess returns is robust. That is, the TIPS mispricing factors predict equity excess returns alone or jointly with inflation and bond excess returns, and their predictive content is stable when combined with the variance risk premium.

In Table 7 we report the results of predictability regressions for one-year excess returns on two- to five-year bonds for different ranks of matrix Π. In Panels B and C, we consider Equations (8) and (9), respectively, in which Z_t contains only one-year excess bond returns, $rx^b_{n,t+12}$, for $n = 2, 3, 4, 5$ years. As in the case of equity returns, we do not reject that $\text{rank}(\Pi) = 3$. The R^2s in Panel B have the same monotonically increasing pattern with those in Panel C of Table 3, and their magnitude increases monotonically with bond maturity. This increase across maturities is on average about 4.5 percentage points. The addition of the variance risk premium in the predictability regressions results in an additional increase of R^2s, which is on average 2.6 percentage points. Therefore, the predictive content of the term structure of TIPS mispricing for bond excess returns is robust.

Finally, the evidence we find for the existence of risk factors that summarize the predictability of inflation, and bond and equity risk premia, either jointly or separately, allows us to distinguish between common factors and idiosyncratic factors. In particular, since there are four factors that summarize the joint predictability of bond and equity risk premia, and the predictability in each risk premium is summarized by three factors, this implies that there are two common factors for bond and equity risk premia, one bond risk premia specific factor, and one equity risk premia specific factor.

7 Conclusion

In this paper, we analyze the term structure of TIPS mispricing. Absence of arbitrage implies that inflation swap rates and break-even inflation rates have to be equal. The data, however, show a consistent positive difference between the two quantities, which we refer to as the TIPS arbitrage mispricing. We show that the term structure of TIPS mispricing, has a low dimensional factor structure, and the factors are interpreted as level, slope, and curvature.

We find that the level factor is a persistent process, with a forward-looking character. We establish a link between volatility in financial markets and the level of TIPS mispricing. The slope factor is a less persistent process and is related to volatility in financial markets, but also to the risk premium that investors demand for bearing this volatility risk. We argue that the slope factor is driven mainly by the slow-moving capital that investors are willing to allocate to TIPS with different maturities in order to hedge against long-run consumption risk.

Finally, we examine the information content of the TIPS mispricing curve in the predictability of inflation, and bond and equity risk premia. The predictability results from the main drivers of the curve: financial volatility and slow-moving capital. We find that that four factors are sufficient to summarize the joint predictability of inflation, and bond and equity excess returns across maturities and across horizons. Furthermore, we find two common factors for bond and equity premia, one bond risk premia specific factor, and one equity risk premia specific factor. The information content of the TIPS mispricing curve is robust when other predictors are included in the analysis.

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Appendix

A Cook-Setodji Test

Cook and Setodji (2003) propose the following estimation algorithm for the rank of a given matrix B:

1. start with $\mathrm{rank}(B) = m = 0$;

2. under the hypothesis $d = m$, compare the Cook-Setodji test statistic, $\widehat{\Lambda}_m$, to the percentage points of a chi-squared distribution and determine the p-value $\hat{p}_m$, which is the probability of exceeding the observed value of $\widehat{\Lambda}_m$;

3. if $\hat{p}_m$ is larger than a selected cutoff, e.g. 5%, then conclude that $\mathrm{rank}(B) = d = m$, that is, there is insufficient information to contradict the hypothesis $d = m$;

4. if it is smaller, then conclude that $d > m$, increment m by 1, and repeat the procedure under the hypothesis $d = m + 1$.

B Reduced Rank Regression

The reduced rank regression (RRR) model can be written as

$$Z_{0,t} = \alpha\beta^\top Z_{1,t} + \Psi Z_{2,t} + \varepsilon_t, \quad t = 1, \dots, T, \tag{1}$$

where $Z_{0,t}$, $Z_{1,t}$, and $Z_{2,t}$ are vectors of dimensions p, p_1, and p_2, respectively, and α, β, and Ψ are parameters of dimensions $p \times r$, $p_1 \times r$, and $p \times p_2$, respectively, and $r < \min(p, q)$. The RRR estimators of α, β, and Ψ are defined as the solution to

$$\min_{\alpha,\beta,\psi} \left\| \sum_{t=1}^{T} \varepsilon_t \varepsilon_t^\top \right\|, \tag{2}$$

and the RRR estimator is the maximum likelihood estimator if ε_t is assumed to be normally distributed. In matrix notation the RRR model takes the form

$$Z_0 = Z_1 \beta \alpha^\top + Z_2 \Psi^\top + \varepsilon, \tag{3}$$

where row t of Z_0, Z_1, Z_2, and ε is $Z_{0,t}^\top$, $Z_{1,t}^\top$, $Z_{2,t}^\top$, and $\varepsilon_t^\top$, respectively, and $Var\left[\varepsilon^\top\right] = I_T \otimes \Omega$. Define the matrices

$$M_{ij} = \frac{Z_i^\top Z_j}{T}, \quad i, j = 0, 1, 2 \quad \text{and} \quad S_{ij} = M_{ij} - M_{i2} M_{22}^{-1} M_{2j}, \quad i, j = 0, 1. \tag{4}$$

The parameter estimators of the RRR model (Hansen, 2008) are given by:

$$\begin{aligned} \hat{\beta}_T &= \begin{pmatrix} \hat{\nu}_{1T} & \hat{\nu}_{2T} & \dots & \hat{\nu}_{rT} \end{pmatrix} \hat{\phi}_T \\ \hat{\alpha}_T &= S_{01} \hat{\beta}_T \left(\hat{\beta}_T^\top S_{11} \hat{\beta}_T \right)^{-1} \\ \hat{\Psi}_T &= \left(M_{02} - \hat{\alpha}_T^\top \hat{\beta}_T M_{12} \right) M_{22}^{-1}, \end{aligned} \tag{5}$$

where $\hat{\nu}_{1T},\ \hat{\nu}_{2T},\ \dots,\ \hat{\nu}_{rT}$ are the eigenvectors corresponding to the r largest eigenvalues $\hat{\lambda}_{1T},\ \hat{\lambda}_{2T},\ \dots,\ \hat{\lambda}_{rT}$ of the generalized eigenvalue problem

$$\left\| \lambda S_{11} - S_{10} S_{00}^{-1} S_{01} \right\|, \tag{6}$$

and $\hat{\phi}_T$ is an arbitrary $r \times r$ matrix with full rank.

Table 1. Summary statistics. Entries are sample moments of daily observations of inflation swap rates, break-even inflation rates, and the difference between the two rates. Mean, median, and standard deviation of rates are expressed as percentages. Mean, median, and standard deviation of the difference are in basis points. The maturities are 2, 3, 4, 5, 10, 15, and 20 years. The sample period is from January 2005 to December 2019.

Panel A: Inflation swap rates							
	2	3	4	5	10	15	20
Mean	1.832	1.958	2.057	2.139	2.393	2.496	2.540
Median	1.854	1.953	2.053	2.156	2.460	2.596	2.653
Std. Dev	0.819	0.653	0.549	0.486	0.357	0.364	0.384
AC(22)	0.886	0.891	0.895	0.892	0.901	0.906	0.911
Skewness	-2.077	-1.545	-1.076	-0.860	-0.480	-0.468	-0.446
Ex. Kurtosis	9.324	6.441	4.105	2.876	-0.569	-0.764	-0.841
% of > 0	97.5	98.3	99.2	99.6	100.0	100.0	100.0
Panel B: Break-even inflation rates							
	2	3	4	5	10	15	20
Mean	1.574	1.677	1.768	1.850	2.129	2.239	2.279
Median	1.668	1.758	1.851	1.939	2.212	2.331	2.368
Std. Dev	0.982	0.810	0.688	0.595	0.420	0.431	0.437
AC(22)	0.901	0.911	0.913	0.911	0.906	0.920	0.921
Skewness	-2.601	-2.486	-2.402	-2.279	-0.868	-0.491	-0.479
Ex. Kurtosis	11.592	10.508	9.813	9.051	1.016	-0.709	-0.526
% of > 0	95.8	96.8	97.3	98.0	100.0	100.0	100.0
Panel C: Difference (TIPS Mispricing)							
	2	3	4	5	10	15	20
Mean	25.724	28.046	28.896	28.879	26.380	25.740	26.078
Median	24.469	23.666	22.994	22.910	25.060	25.348	25.344
Std. Dev	24.973	24.719	25.264	23.497	13.139	11.588	11.994
AC(22)	0.822	0.893	0.878	0.877	0.759	0.681	0.691
Skewness	2.455	2.917	3.397	3.372	3.443	1.332	1.480
Ex. Kurtosis	11.598	12.277	16.545	16.319	16.723	5.349	7.044
% of Obs > 0	90.3	97.5	98.5	99.6	100.0	99.8	99.3

Table 2. Correlation matrix. Entries are sample correlations computed from daily observations. The monthly realized variance measure, measured at daily frequency, is the sum of squared 5-min returns over a 22-day period (RV). The expected realized variance measure is a statistical forecast of monthly realized variance, $RVFor = \widehat{E}_t\left[RV_{t,t+22}\right]$, using the Heterogeneous Autoregressive model of Realized Variance (HAR-RV). The risk-neutral expectation of monthly realized variance is the option-implied variance and is measured as the de-annualized VIX-squared ($\text{VIX}^2 30/365$). The daily variance risk premium (VRP) is the difference between the option-implied variance and the expected realized variance measure. PC1, PC2, and PC3 are the first three principal components scores which are computed at a daily frequency via an eigenvalue decomposition of the correlation matrix of the difference between the inflation swap rate and the break-even inflation rate. Level is the average of $d_{n,t}$ across maturities, slope is $d_{20,t} - d_{2,t}$, and curvature is $d_{20,t} + d_{2,t} - 2d_{10,t}$, where $d_{n,t}$ is the difference between the n-year inflation swap rate and the n-year break-even inflation rate. The sample period is from January 2005 to December 2019.

	VIX^2	RVFor	RV	VIX	VRP	PC1	PC2	PC3	Level	Slope	Curv.
VIX^2	1.000										
Forecasted RV	0.905	1.000									
RV	0.919	0.990	1.000								
VIX	0.954	0.841	0.851	1.000							
VRP	0.578	0.176	0.227	0.594	1.000						
PC1	0.746	0.727	0.703	0.719	0.331	1.000					
PC2	-0.303	-0.309	-0.337	-0.307	-0.107	0.000	1.000				
PC3	-0.102	-0.143	-0.109	-0.101	0.038	0.000	0.000	1.000			
Level	0.770	0.751	0.730	0.743	0.342	0.995	-0.095	0.018	1.000		
Slope	-0.625	-0.617	-0.626	-0.595	-0.262	-0.629	0.629	-0.338	-0.691	1.000	
Curvature	0.333	0.284	0.318	0.329	0.226	0.400	-0.349	0.797	0.447	-0.666	1.000

Table 3. Inflation, excess returns, and the term structure of TIPS mispricing. Entries are rank test p-values and adjusted R^2s of multivariate regressions, $Z_t = \alpha + \Pi D_t + \epsilon_t$, based on monthly observations, where Z_t contains one-year ahead inflation, π_{t+12}, bond excess returns $rx^b_{n,t+12}$, where $n = 2, 3, 4, 5$ years, and equity excess returns, rx^e_{t+h}, where $h = 1, 2, 3, 6, 9, 12$ months, and $D_t = \{d_{n,t}\}_{n=1,\ldots,p_1}$ is a vector of dimension p_1 with the difference between inflation swap rates and break-even inflation rates at all available maturities. Panel A reports the p-values of the Cook-Setodji test statistic, $\widehat{\Lambda}_m$, which tests the null hypothesis H_0 that the rank of the matrix Π is r for ranks ranging from 0 to 10. Panel B reports the R^2s of predictability regressions of one-year ahead realized inflation obtained via multivariate RRR estimation for different ranks of matrix Π. Panel C reports the R^2s of predictability regressions of bond excess returns. Panel D reports the R^2s of predictability regressions of equity excess returns. The sample period is from January 2005 to December 2019.

Panel A: Rank test p-values							
H_0 :	$r=0$	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$
p-value	0.0	0.0	0.0	0.0	5.4	13.5	26.5
Panel B: Inflation adj. R^2s							
	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$	$r=7$
$\pi_{t,t+12}$	11.5	14.3	15.1	16.1	17.1	16.6	16.6
Panel C: Bond returns adj. R^2s							
	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$	$r=7$
2	10.3	19.0	18.5	22.3	23.2	22.7	22.3
3	15.4	22.5	22.0	25.3	26.4	26.0	25.5
4	19.5	25.6	25.2	27.6	28.8	28.5	28.1
5	22.6	27.9	27.6	29.3	30.5	30.3	29.9
Panel D: Equity returns adj. R^2s							
	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$	$r=7$
1	8.70	14.2	14.9	15.0	14.6	14.9	14.8
2	12.6	20.4	19.9	21.1	20.7	20.6	21.1
3	15.5	22.3	25.0	25.2	25.2	25.3	25.2
6	27.2	28.6	28.4	28.7	28.3	29.1	28.9
9	26.8	26.5	26.6	26.9	27.0	27.9	27.5
12	18.6	18.3	18.0	18.0	18.9	19.8	19.4

Table 4. Excess equity returns and the variance risk premium. Entries are slope coefficients and adjusted R^2s of the regression, $rx^e_{t+h} = \alpha_h + \beta_h vrp_t + \epsilon_{t+h}$, based on monthly observations, where rx^e_{t+h}, where $h = 1, 2, 3, 6, 9, 12$ months, are equity excess returns, and vrp_t is the monthly variance risk premium, which is the difference between the option-implied variance and the lagged monthly realized variance measure. The option-implied variance is a proxy for the risk-neutral expectation of monthly realized variance at month t, and is measured as the end-of-month, de-annualized VIX-squared (VIX^{2}30/365). The lagged monthly realized variance measure is the sum of daily realized variances over the month $t-1$, and the daily realized variance for a given day is the sum of squared 5-min returns for that day.

Main Sample: January 2005 to December 2019						
	1	2	3	6	9	12
$\hat{\beta}_h$	5.13	4.01	5.13	2.74	1.45	1.10
$se(\hat{\beta}_h)$	2.15	1.37	1.08	1.03	0.99	0.79
Adj. R^2_h	4.15	4.58	11.78	5.37	1.90	1.34
Full Sample: January 1990 to March 2015						
	1	2	3	6	9	12
$\hat{\beta}_h$	4.63	3.98	4.25	2.73	1.59	1.25
$se(\hat{\beta}_h)$	1.31	0.95	0.71	0.63	0.55	0.48
Adj. R^2_h	3.90	5.43	9.26	7.04	3.44	2.64
Subsample: January 1990 to December 2012						
	1	2	3	6	9	12
$\hat{\beta}_h$	4.84	4.19	4.46	2.93	1.75	1.37
$se(\hat{\beta}_h)$	1.32	1.00	0.75	0.66	0.56	0.49
Adj. R^2_h	4.31	6.00	10.19	8.12	4.20	3.24
Subsample: January 1990 to October 2007						
	1	2	3	6	9	12
$\hat{\beta}_h$	2.69	4.34	3.73	2.31	1.45	1.13
$se(\hat{\beta}_h)$	1.82	1.22	1.01	0.84	0.91	0.86
Adj. R^2_h	0.55	4.70	5.21	4.22	2.25	1.44

Table 5. Excess equity returns and the term structure of TIPS mispricing. Entries are rank test p-values and adjusted R^2s of multivariate regressions based on monthly observations. Panel B corresponds to $Z_t = \alpha + \Pi D_t + \epsilon_t$, and Panel C to $Z_t = \alpha + \Pi D_t + \psi vrp_t + \epsilon_t$, where Z_t contains equity excess returns, rx^e_{t+h}, where $h = 1, 2, 3, 6, 9, 12$ months, $D_t = \{d_{n,t}\}_{n=1,\ldots,p_1}$ is a vector of dimension p_1 with the difference between inflation swap rates and break-even inflation rates at all available maturities, and vrp_t is a scalar with the monthly variance risk premium. Panel A reports the p-values of the Cook-Setodji test statistic, $\widehat{\Lambda}_m$, which tests the null hypothesis H_0 that the rank of the matrix Π is r for ranks ranging from 0 to 5. Panels B and C report the R^2s of predictability regressions of equity excess returns obtained via multivariate RRR estimation for different ranks of matrix Π. The sample period is from January 2005 to December 2019.

Panel A: Rank test p-values						
H_0 :	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
p-value	0.0	0.1	9.6	77.4	87.5	98.9

Panel B: Adj. R^2s - TIPS mispricing						
	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$	$r = 6$
1	13.4	14.0	16.5	16.3	15.8	15.3
2	20.0	19.5	22.9	22.5	22.0	21.6
3	23.6	26.3	27.0	26.6	26.1	25.7
6	31.3	30.8	30.4	30.2	29.8	29.4
9	28.2	27.9	29.2	28.8	28.4	27.9
12	19.9	19.7	20.9	20.8	20.3	19.9

Panel C: Adj. R^2s - TIPS mispricing and variance risk premium						
	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$	$r = 6$
1	20.9	23.7	24.4	24.2	23.7	23.2
2	25.0	24.9	27.3	27.1	26.7	26.3
3	32.6	33.1	35.4	35.0	34.6	34.2
6	34.7	34.5	34.2	34.1	33.8	33.4
9	29.5	29.9	31.0	30.6	30.2	29.8
12	22.3	22.2	23.1	23.1	22.7	22.2

Table 6. Excess bond returns and the Cochrane-Piazzesi factor.

Main Sample: January 2005 to December 2019							
	2		3		4		5
b_n	0.34		0.78		1.23		1.65
Adj. R_n^2	28.00		33.96		38.74		41.95
	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	Adj. R^2
OLS estimates	-2.73	1.69	-3.03	-2.22	5.82	-1.24	36.97

Subsample: January 1964 to March 2015							
	2		3		4		5
b_n	0.45		0.85		1.25		1.45
Adj. R_n^2	19.06		21.05		24.54		22.51
	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	Adj. R^2
OLS estimates	-1.47	-1.29	-0.58	1.78	1.34	-1.05	22.13

Subsample: January 1964 to December 2003							
	2		3		4		5
b_n	0.47		0.87		1.24		1.43
Adj. R_n^2	30.52		33.17		36.47		33.82
	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	Adj. R^2
OLS estimates	-3.32	-2.06	0.65	3.03	0.80	-2.01	33.90

Table 7. Excess bond returns and the term structure of TIPS mispricing. Entries are rank test p-values and adjusted R^2s of multivariate regressions based on monthly observations. Panel B corresponds to $Z_t = \alpha + \Pi D_t + \epsilon_t$, and Panel C to $Z_t = \alpha + \Pi D_t + \psi vrp_t + \epsilon_t$, where Z_t contains bond excess returns $rx^b_{n,t+12}$, where $n = 2, 3, 4, 5$ years, $D_t = \{d_{n,t}\}_{n=1,\dots,p_1}$ is a vector of dimension p_1 with the difference between inflation swap rates and break-even inflation rates at all available maturities, and vrp_t is a scalar with the monthly variance risk premium. Panel A reports the p-values of the Cook-Setodji test statistic, $\widehat{\Lambda}_m$, which tests the null hypothesis H_0 that the rank of the matrix Π is r for ranks ranging from 0 to 3. Panels B and C report the R^2s of predictability regressions of bond excess returns obtained via multivariate RRR estimation for different ranks of matrix Π. The sample period is from January 2005 to December 2019.

Panel A: Rank test p-values				
H_0 :	$r = 0$	$r = 1$	$r = 2$	$r = 3$
p-value	0.0	0.0	0.2	34.8
Panel B: Adj. R^2s - TIPS mispricing				
	$r = 1$	$r = 2$	$r = 3$	$r = 4$
2	19.9	23.6	23.8	23.7
3	25.1	27.3	27.0	26.9
4	28.6	30.0	29.6	29.4
5	30.8	31.7	31.5	31.2
Panel C: Adj. R^2s - TIPS mispricing and variance risk premium				
	$r = 1$	$r = 2$	$r = 3$	$r = 4$
2	20.7	25.6	25.7	25.4
3	27.2	30.2	29.9	29.6
4	31.4	33.5	33.1	32.8
5	33.8	35.2	35.0	34.7
Panel D: Adj. R^2s - TIPS mispricing and Cochrane-Piazzesi factor				
	$r = 1$	$r = 2$	$r = 3$	$r = 4$
2	28.0	28.9	29.7	30.2
3	35.0	35.1	35.8	36.3
4	41.2	40.9	41.6	41.9
5	45.7	45.3	46.1	46.3

Figure 1. Main time series: inflation swap rates, break-even inflation rates, and difference. The lines in the top panel represent inflation swap rates, and in the middle panel break-even inflation rates. The lines in the bottom panel represent differences between the inflation swap rate and the break-even inflation rate. Inflation swap rates and break-even inflation rates are continuously compounded rates for two-year, five-year, ten-year, and twenty-year maturities over the period from January 2005 to December 2019. Rates are expressed as annual percentages, and the difference is in annual basis points.

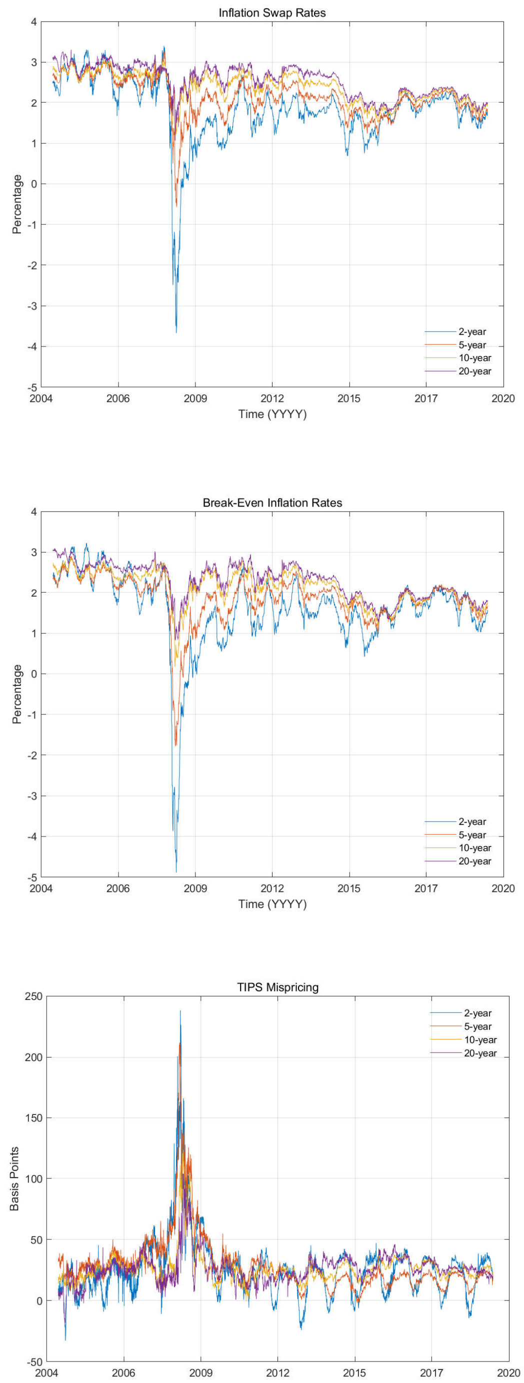

Figure 2. Term structures of mean, median, and standard deviation of main time series.

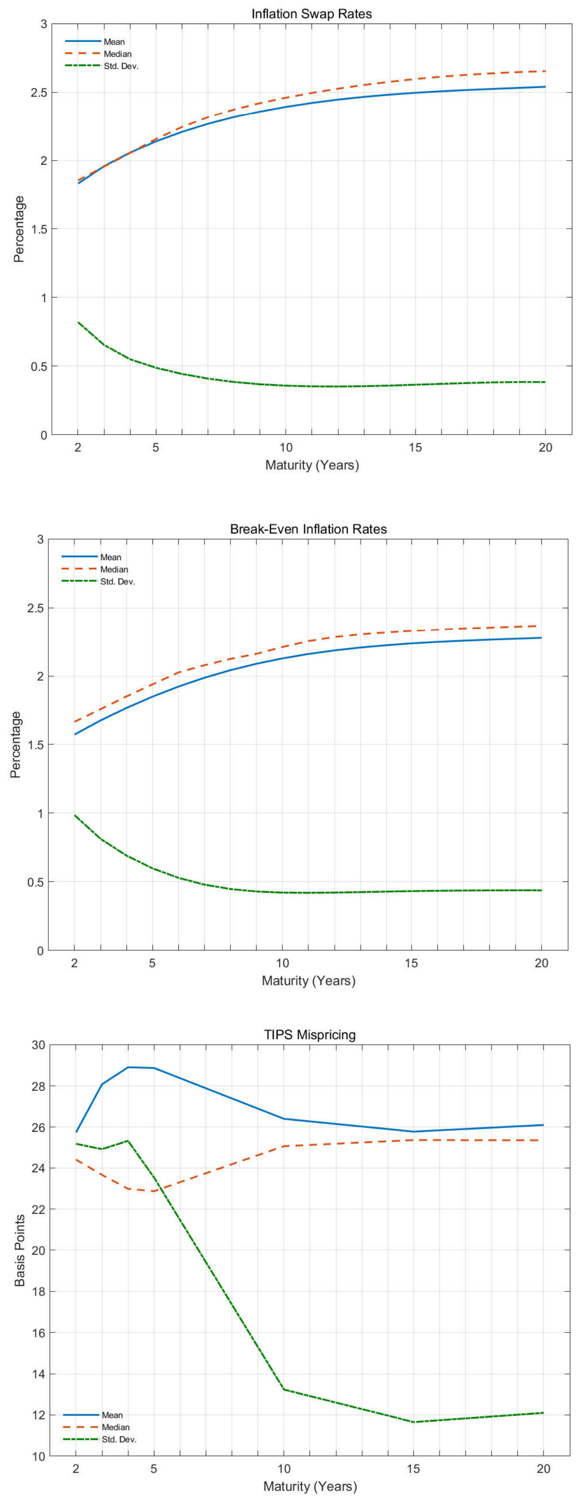

Figure 3. Principal component analysis. The principal components are computed at a daily frequency via an eigenvalue decomposition of the correlation matrix of the difference between the inflation swap rate and the break-even inflation rate over the period from January 2005 to December 2019. The top left panel displays the loadings for the first three principal components, the top right panel displays the scores, and the bottom panel reports the variance explained (%) by the principal components.

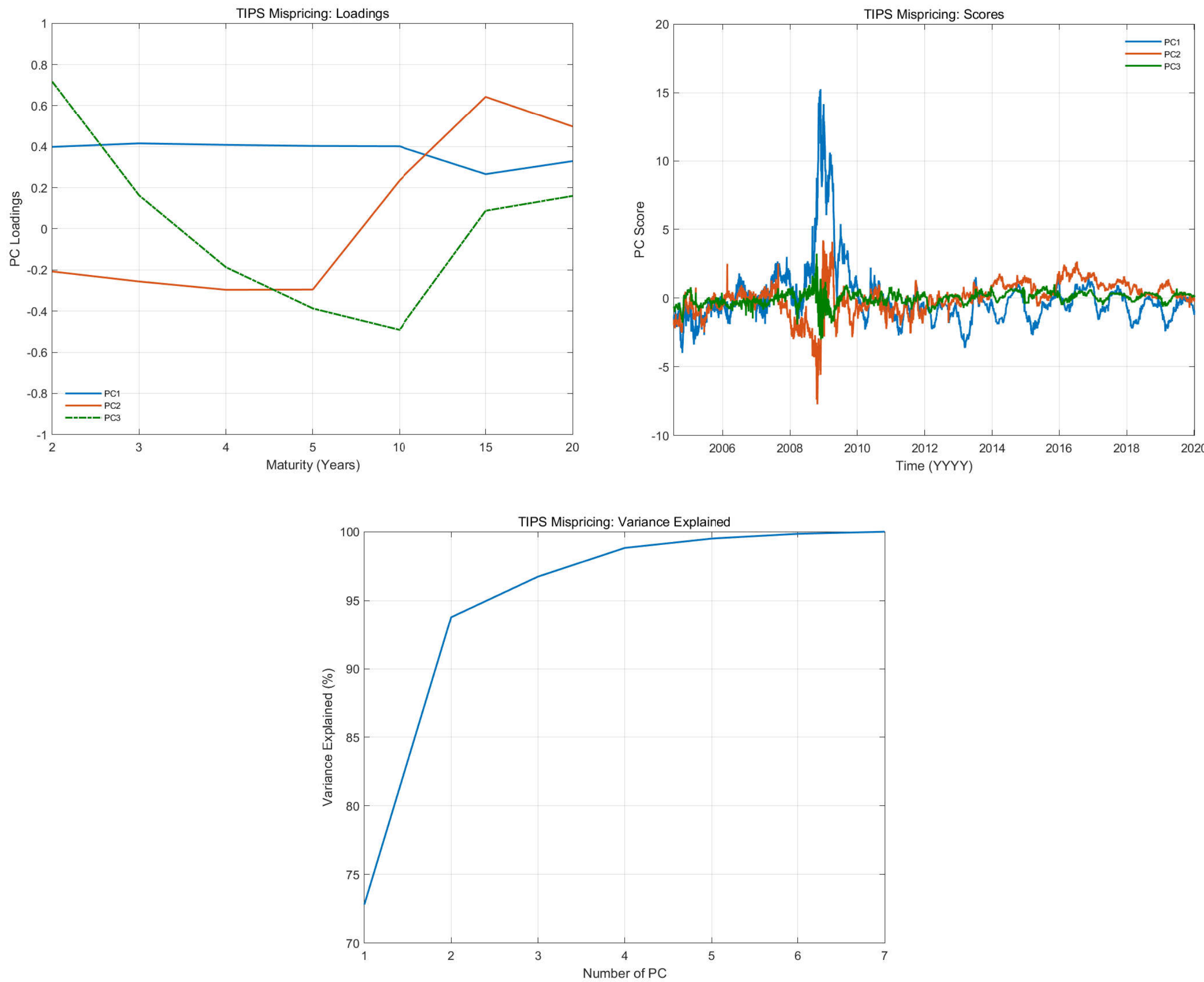

Figure 4. Difference between inflation swap rates and break-even inflation rates, and VIX. In the top panels the blue lines represent differences between the inflation swap rate and the break-even inflation rate for a two-year (left panel) and a five-year (right panel) maturity, expressed in annual basis points (left y-axis), and the grey line represents VIX expressed as an annual percentage (right y-axis), over the period from January 2005 to December 2019. The bottom panels plot the cross-correlation and autocorrelation functions for the difference between the inflation swap rate and the break-even inflation rate, and VIX.

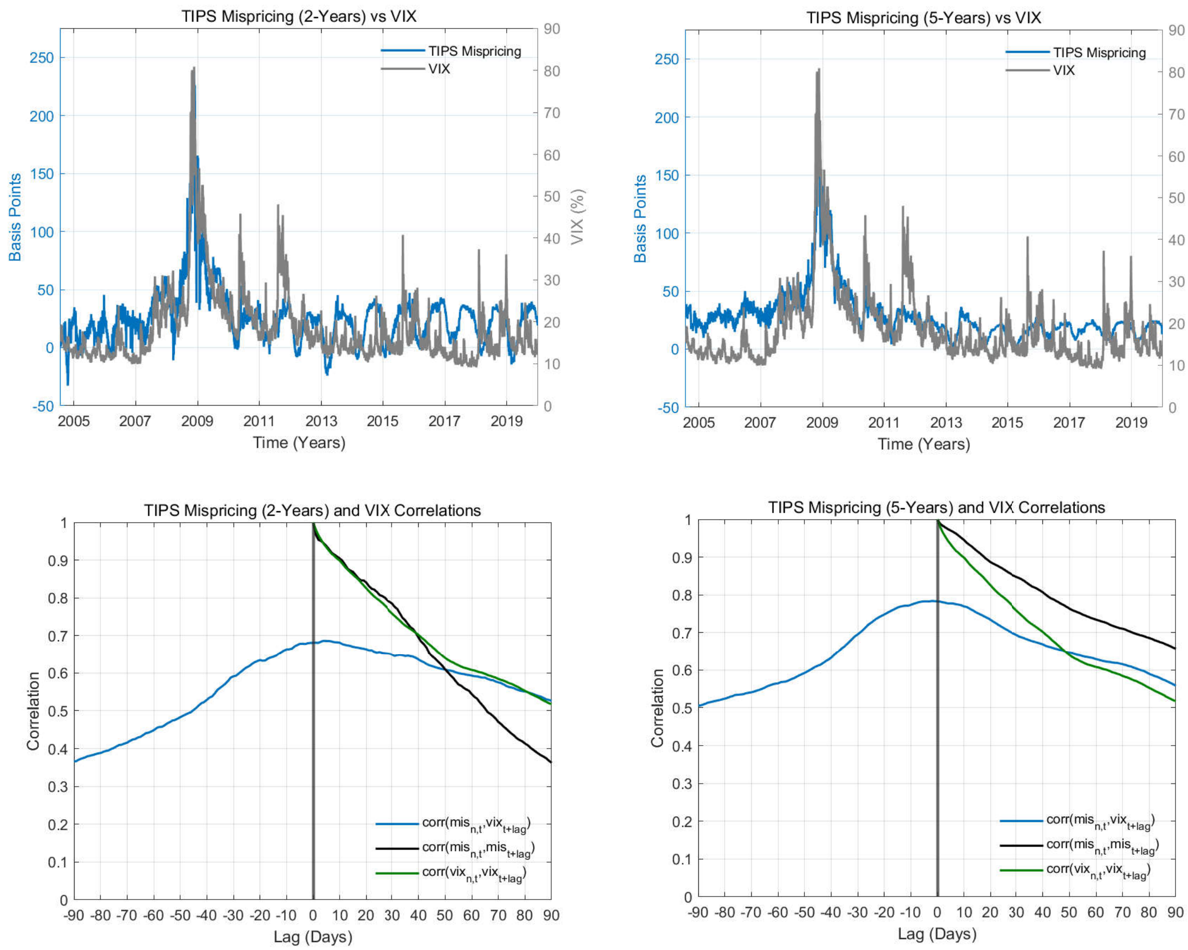

Figure 5. Variance risk premium. The lines in the left panel show the time series of option-implied variance (VIX^2) and expected realized variance measure (RVFor). The lines in the right panel represent variance risk premia. The monthly realized variance measure, measured at daily frequency, is the sum of squared 5-min returns over a 22-day period (RV). The expected realized variance measure is a statistical forecast of monthly realized variance, $RVFor = \widehat{E}_t\left[RV_{t,t+22}\right]$, using the Heterogeneous Autoregressive model of Realized Variance (HAR-RV). The risk-neutral expectation of monthly realized variance is the option-implied variance and is measured as the de-annualized VIX-squared ($VIX^2 30/365$). The daily variance risk premium (VRP) is the difference between the option-implied variance and the expected realized variance measure. The sample period is from January 2005 to December 2019. Variance measure is in monthly terms, and its square root multiplied by $\sqrt{365/30}$ is expressed as an annual percentage. Variance risk premium is in monthly basis points.

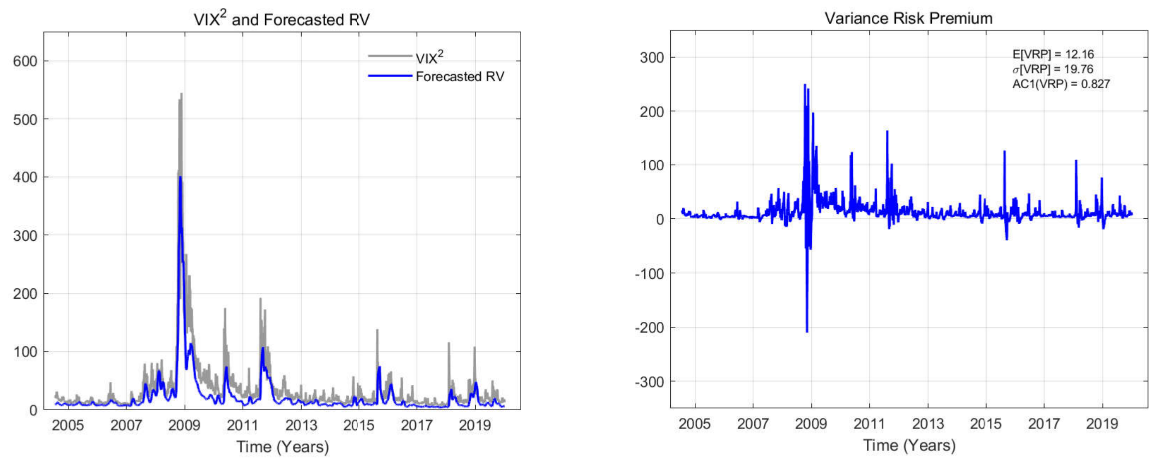

Figure 6. Autocorrelation. The lines represent autocorrelation functions. The monthly realized variance measure, measured at daily frequency, is the sum of squared 5-min returns over a 22-day period (RV). The expected realized variance measure is a statistical forecast of monthly realized variance, $RVFor = \widehat{E}_t\left[RV_{t,t+22}\right]$, using the Heterogeneous Autoregressive model of Realized Variance (HAR-RV). The risk-neutral expectation of monthly realized variance is the option-implied variance and is measured as the de-annualized VIX-squared ($\text{VIX}^2 30/365$). The daily variance risk premium (VRP) is the difference between the option-implied variance and the expected realized variance measure. PC1, PC2, and PC3 are the first three principal components scores which are computed at a daily frequency via an eigenvalue decomposition of the correlation matrix of the difference between the inflation swap rate and the break-even inflation rate. The sample period is from January 2005 to December 2019.

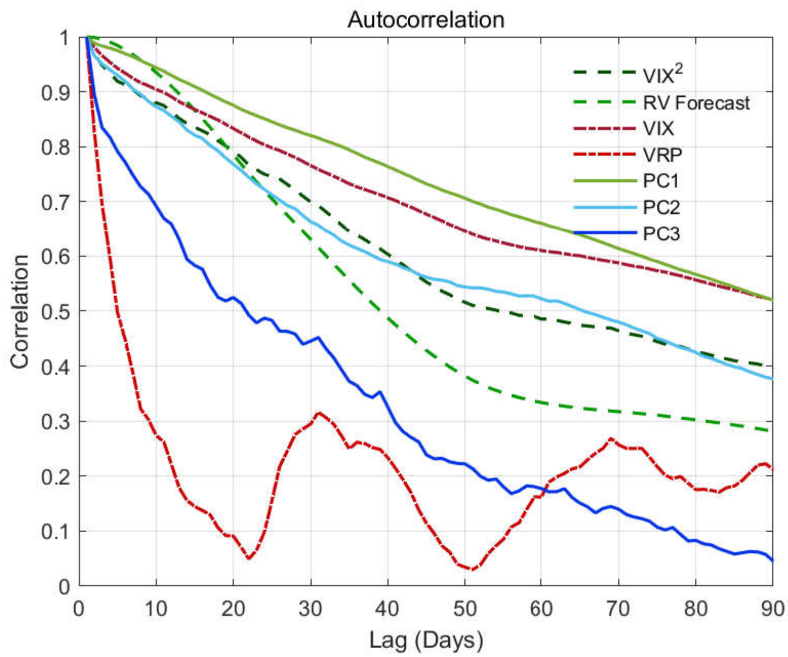

Figure 7. Cross-correlation. The lines represent cross-correlation functions. The top panels show the sample cross-correlation between option-implied variance, variance risk premia, principal components scores and leads and lags of squared daily log returns of the S&P 500 index ranging from -22 to 22 days. The other panels show the sample cross-correlation between principal components scores and leads and lags of option-implied volatility, option-implied variance, and variance risk premium ranging from -90 to 90 days. The sample period is from January 2005 to December 2019.

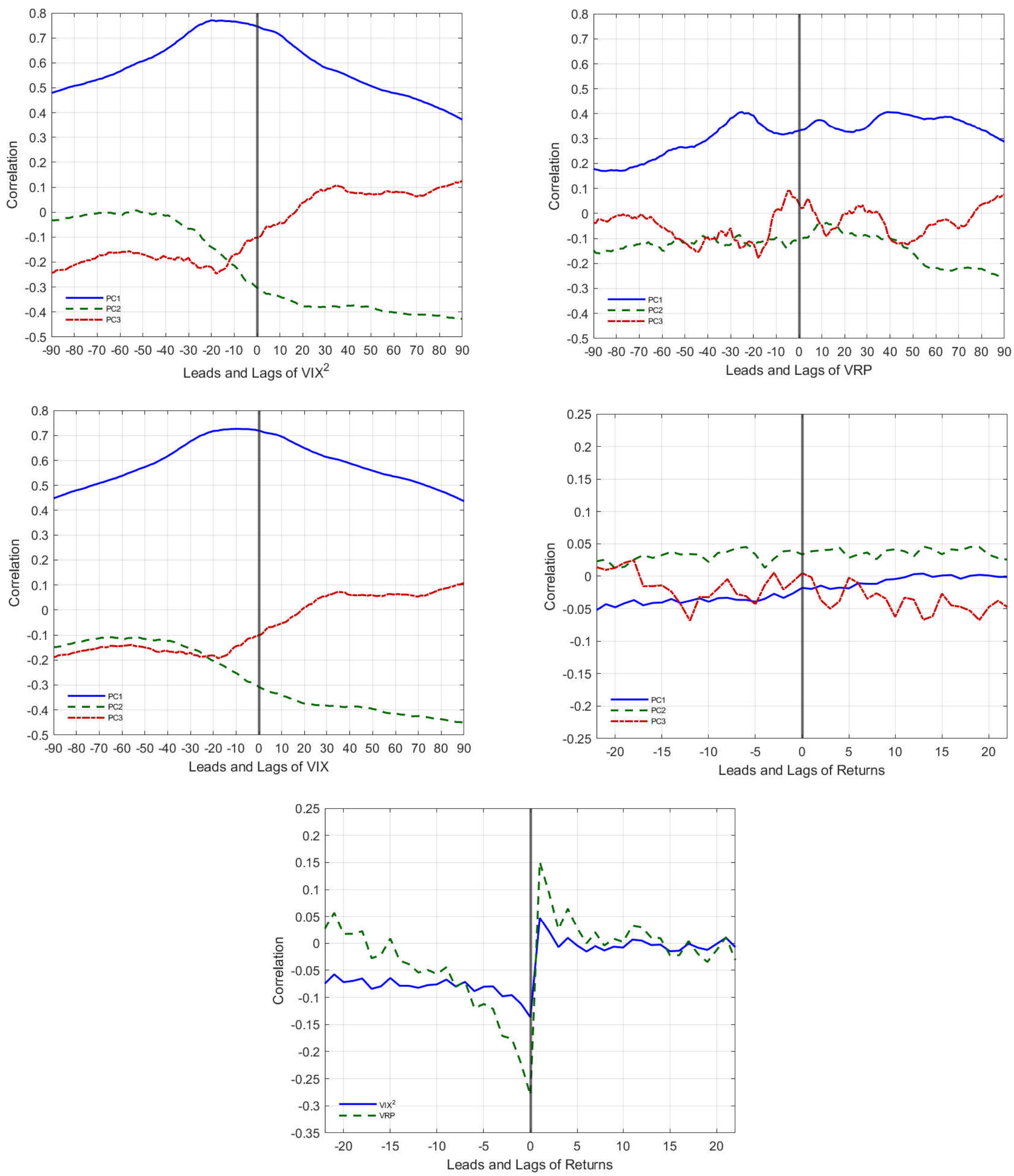

Figure 8. Principal components. The red lines represent principal components scores. The grey lines represent option-implied variance, variance risk premium, monthly realized variance computed as the sum of squared 5-min returns of the S&P 500 index over a 22-day period, and its expected realized variance, computed using the Heterogeneous Autoregressive model of Realized Variance (HAR-RV). The sample period is from January 2005 to December 2019.

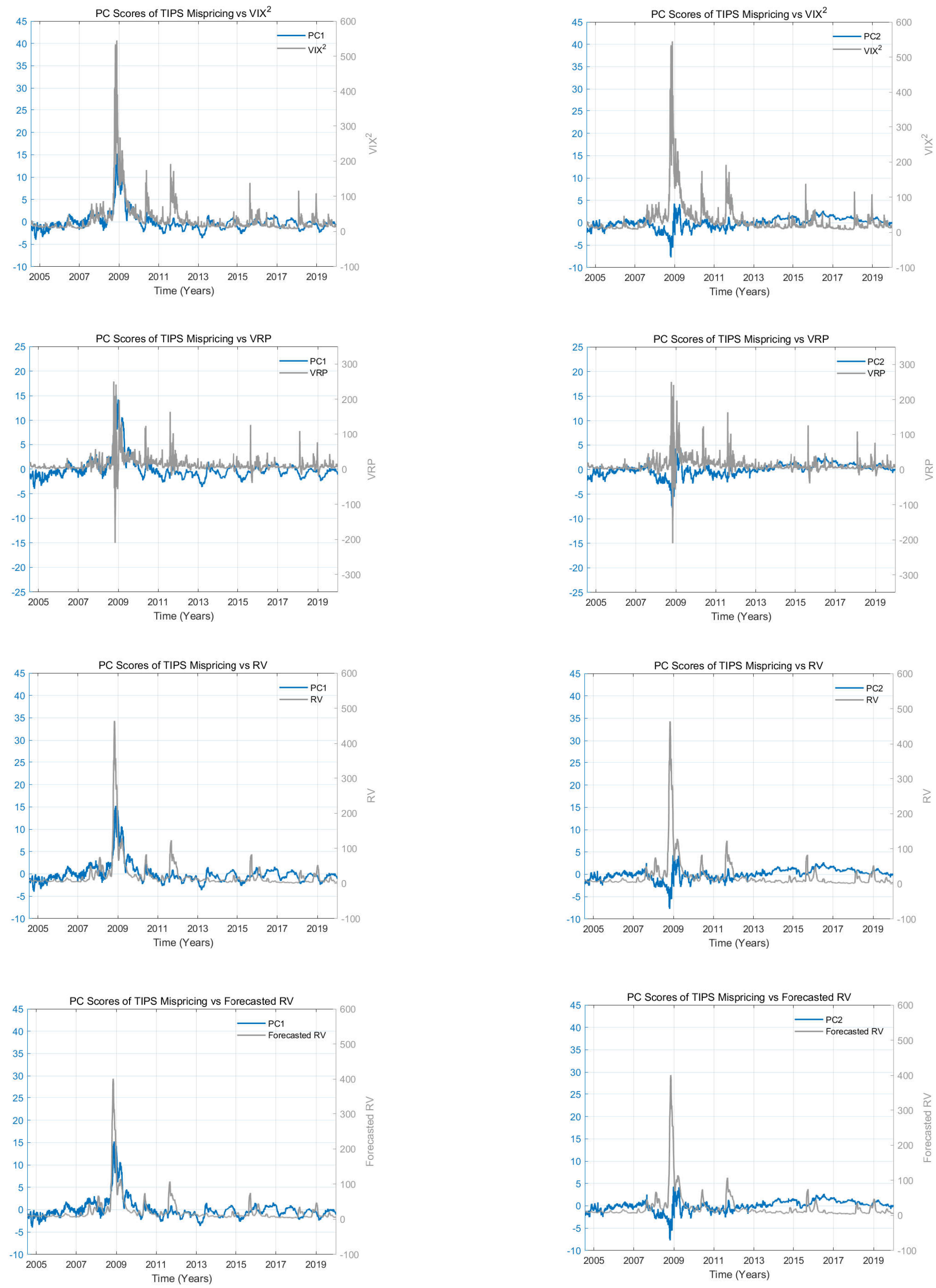

Figure 9. Average yield curves. The lines represent mean zero-coupon yields on nominal U.S. Treasury bonds (light blue) and TIPS (red), and mean differences between zero-coupon yields on nominal U.S. Treasury bonds and inflation swap rates of the same maturity (dark blue) which is an alternative proxy for the real yield curve. The sample period is from January 2005 to December 2019. Bond yields and swap rates are continuously compounded and expressed as annual percentages.